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Spot.	First seen.	Last seen.	No. of Rotations.	Rotation Period. h	m	s
1897 A	June 13	August 19	157 *	10	14	0·6
„ B	July 10	July 18	19	10	13	23·3
„ C	„ 10	„ 16	14	10	14	15·2
„ D	„ 10	„ 16	14	10	14	36·6
	Mean period of rotation		... ..	10	14	4·

It became increasingly difficult to see the spots, as the southern declination of *Saturn* increased in 1898, 1899, and 1900, and the rarity with which they were seen stood in the way of any serious identification of them. The results, however, for 1896 and 1897 are satisfactory for +18° of kronocentric latitude, and in fair agreement with the work of other observers.

On the Accuracy of Eye-observations of Meteors and the Determination of their Radiant Point. By Bryan Cookson.

The probable existence of stationary radiants for so many meteoric showers and the difficulty of finding a satisfactory physical explanation of them, makes the question of the determination of the radiant of considerable importance. The following is an attempt to obtain a numerical estimate of the accuracy of eye-observations of meteors. So far only *Perseids* have been used for the purpose, this being the shower which afforded the largest number of bright and easily observed meteors; but it would be interesting to apply the method to other showers with different characteristics, such as those of April and October.

A chart of the celestial sphere on the gnomonic projection was drawn on a scale of about $\frac{1}{10}$-inch to 1° R.A., and $\frac{1}{5}$-inch to 1° Decl., with centre at R.A. 45°, Decl. +57°, which is the assumed position of the *Perseid* radiant on August 10. From the recorded R.A. and Decl. of beginning and end, the paths of the meteors were drawn upon tracing paper placed on the chart, and the equation of the line of the meteor referred to rectangular coordinates, with their origin at R.A. 45°, Decl. +57, was found graphically by an easy and rapid method in the form

$$x \cos \alpha + y \sin \alpha - p = 0.$$

The radiant is the point such that the sum of the squares of the perpendiculars from it to these lines is a minimum. We have, therefore, to solve by least squares a system of equations of the above form for x and y, which are small corrections to be applied to the approximate radiant.

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* The identity of the spots seen on 1897 June 13 and August 19 is somewhat doubtful, as there are no intermediate observations.

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Weights have been assigned to the equations based upon the following considerations. It is assumed that for a given observer an error in the recorded beginning or end of a meteor's path is equally probable in any direction. This is not, however, strictly true: Professor Weiss found that the probable error of a recorded beginning or end was about ±2°·95, but that the direction of path was more accurate than this large error would lead one to expect. This is owing to the fact that many observers record the end of a trail in the prolongation of the meteor's path, and hence by comparing observations of the same meteors by different observers the differences in the celestial coordinates of the end may be considerable, whilst the direction of the path may be very nearly the same for all. The probable error as far as direction is concerned was found to be ±1°·0 (*Sitzungsber. Wien Akad.* vol. 62 Abt. II. pp. 277–344). As we are here only concerned with direction, it seems legitimate to make the above assumption.

Let then a small arc ρ of great circle be resolved into two perpendicular components: to find the mean error of these components ϵ_1 and ϵ_2, corresponding to a mean error ϵ of ρ, we have

$$\epsilon_1^2=\frac{\epsilon^2}{2\pi}\int_0^{2\pi}\cos^2\psi d\psi=\frac{\epsilon^2}{2}\ :\ \epsilon_2^2=\frac{\epsilon^2}{2\pi}\int_0^{2\pi}\sin^2\psi d\psi=\frac{\epsilon^2}{2}.$$

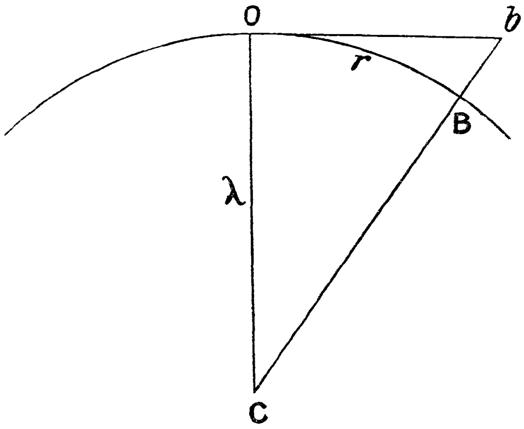
The mean error of the components is therefore $\frac{\epsilon}{\sqrt{2}}$.

If now B be projected into b, the coordinates of b are

$$x=\lambda\tan r\cos\theta$$
$$y=\lambda\tan r\sin\theta.$$

The small displacements of B in perpendicular directions are clearly

$$dr \text{ and } \sin r\,.\,d\theta,$$



and we have just seen that the mean error of each is $\epsilon/\sqrt{2}$.

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But it is known that if a linear differential relation such as

$$dP = A\,dx + B\,dy + C\,dz + \ldots$$

holds good, then

$$\epsilon_P^2 = A^2\epsilon_x^2 + B^2\epsilon_y^2 + C^2\epsilon_z^2 + \ldots$$

where ϵ_P denotes the mean error of P, &c. In our case we have

$$dx = \lambda \sec^2 r \cos\theta\, dr - \lambda \tan r \sin\theta \frac{1}{\sin r} (\sin r\, d\theta),$$

and therefore

$$\epsilon_x^2 = \lambda^2 \sec^4 r \cos^2\theta \frac{\epsilon^2}{2} + \lambda^2 \tan^2 r \sin^2\theta \frac{1}{\sin^2 r}\frac{\epsilon^2}{2}$$

$$= \frac{\epsilon^2}{2} \lambda^2 \sec^2 r \left[(1 + \tan^2 r)\cos^2\theta + \sin^2\theta\right]$$

$$= \frac{\epsilon^2}{2} (\lambda^2 + x^2 + y^2)\left(1 + \frac{x^2}{\lambda^2}\right)$$

$$= \frac{\epsilon^2}{2} \frac{(\lambda^2 + \rho^2)(\lambda^2 + x^2)}{\lambda^2}$$

Similarly,

$$dy = \lambda \sec^2 r \sin\theta\, dr + \lambda \tan r \cos\theta \frac{1}{\sin r} (\sin r\, d\theta)$$

and

$$\epsilon_y^2 = \frac{\epsilon^2}{2} \lambda^2 \sec^2 r \left[(1 + \tan^2 r)\sin^2\theta + \cos^2\theta\right]$$

$$= \frac{\epsilon^2}{2} \frac{(\lambda^2 + \rho^2)(\lambda^2 + y^2)}{\lambda^2}.$$

I am indebted to Professor Turner for pointing out this method of deducing the expressions for ϵ_x and ϵ_y; it leads to the same results but is much shorter and more elegant than that originally used by me.

Now, since in the equations of condition $x \cos\alpha + y \sin\alpha - p = 0$ x and y are small quantities, we may neglect the mean errors of $\cos\alpha$ and $\sin\alpha$, and consider the weight of the equation as proportional to the square of the mean error of the absolute term, p.

Let, therefore, $x_1 y_1$ be the coordinates of the beginning of the meteor's path, and $x_2 y_2$ those of the end.

Then

$$p = \frac{y_2 x_1 - y_1 x_2}{\left\{\overline{x_2 - x_1}^2 + \overline{y_2 - y_1}^2\right\}^{\frac{1}{2}}} = \frac{y_2 x_1 - y_1 x_2}{D}$$

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$$\therefore\ dp = \frac{1}{D}\left\{ y_2 + \frac{p}{D}(x_2 - x_1) \right\} dx_1 - \frac{1}{D}\left\{ y_1 + \frac{p}{D}\overline{x_2 - x_1} \right\} dx_2$$

$$- \frac{1}{D}\left\{ x_2 - \frac{p}{D}(y_2 - y_1) \right\} dy_1 + \frac{1}{D}\left\{ x_1 - \frac{p}{D}\overline{y_2 - y_1} \right\} dy_2$$

$$= \frac{1}{D}\left\{ y_2 - p \sin \alpha \right\} dx_1 \quad - \frac{1}{D}\left\{ y_1 - p \sin \alpha \right\} dx_2$$

$$- \frac{1}{D}\left\{ x_2 - p \cos \alpha \right\} dy_1 \quad + \frac{1}{D}\left\{ x_1 - p \cos \alpha \right\} dy_2$$

$$= - \frac{1}{D}\sqrt{\rho_2{}^2 - p^2}\ .\ \cos \alpha\ .\ dx_1 + \frac{1}{D}\sqrt{\rho_1 - p^2}\ .\ \cos \alpha\ dx_2$$

$$- \frac{1}{D}\sqrt{\rho_2{}^2 - p^2}\ \ \sin \alpha\ .\ dy_1 + \frac{1}{D}\sqrt{\rho^2 - p^2}\ .\ \sin \alpha\ .\ dy_2.$$

Hence,

$$\varepsilon_p{}^2 = \frac{1}{D^2}\left[\cos^2 \alpha \left\{(\rho_2{}^2 - p^2)\varepsilon^2{}_{x_1} + (\rho_1{}^2 - p^2)\varepsilon^2{}_{x_2}\right\} + \sin^2 \alpha \left\{(\rho_2{}^2 - p^2)\varepsilon^2{}_{y_1} + (\rho_1{}^2 - p^2)\varepsilon^2{}_{y_2}\right\}\right]$$

or

$$\varepsilon_p{}^2 = \frac{\varepsilon^2}{2}\ \frac{1}{\lambda^2 D^2}\left[(\lambda^2 + \rho_1{}^2)(\rho_2{}^2 - p^2)(\lambda^2 + x_1{}^2 \cos^2 \alpha + y_1{}^2 \sin^2 \alpha) + (\lambda^2 + \rho_2{}^2)(\rho_1{}^2 - p^2)(\lambda^2 + x_2{}^2 \cos^2 \alpha + y_2{}^2 \sin^2 \alpha)\right]$$

$$= \frac{\varepsilon^2}{2}\ \frac{1}{\lambda^2 D^2}\left[(\lambda^2 + p^2)\left\{(\lambda^2 + \rho_1{}^2)(\rho_2{}^2 - p^2) + (\lambda^2 + \rho_2{}^2)(\rho_1{}^2 - p^2)\right\} - 2 \sin \alpha \cos \alpha \left\{x_1 y_1(\lambda^2 + \rho_1{}^2)(\rho_2{}^2 - p^2) + x_2 y_2(\lambda^2 + \rho_2{}^2)(\rho_1{}^2 - p^2)\right\}\right]$$

$$= \frac{\varepsilon^2}{2}\ \frac{1}{\lambda^2 D^2}\left[(\lambda^2 + p^2)P - 2 \sin \alpha \cos \alpha\ Q\right], \text{ suppose.}$$

The scale of projection used is such that $\lambda = 10$ inches; the mean value of p is then about ·5, and of ρ_1 about 3; hence, excepting in a very few cases where ρ_1 is small (< 1), that is when the meteor starts from very near the radiant, it is safe to neglect p^2 and take

$$P = \rho_1{}^2(\lambda^2 + \rho_2{}^2) + \rho_2{}^2(\lambda^2 + \rho_1{}^2).$$

If $\theta_1 \theta_2$ are the angles which the lines joining the beginning $x_1 y_1$ and end $x_2 y_2$ of the meteor's path make with the axis of x, then

$$\frac{x_1 y_1}{\rho_1{}^2} = \frac{1}{2} \sin 2\theta_1 \qquad \frac{x_2 y_2}{\rho_2{}^2} = \frac{1}{2} \sin^2 \theta_2$$

$$\therefore\ \frac{x_1 y_1}{\rho_1{}^2} - \frac{x_2 y_2}{\rho_2{}^2} = \sin(\theta_1 - \theta_2) \cos(\theta_1 + \theta_2)$$

$$= \left(\frac{y_1}{\rho_1}\frac{x_2}{\rho_2} - \frac{y_2}{\rho_2}\frac{x_1}{\rho_1}\right) \cos(\theta_1 + \theta_2)$$

$$= - \frac{pD}{\rho_1 \rho_2} \cos(\theta_1 + \theta_2)$$

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Thus

$$Q = x_1 y_1 \left[(\lambda^2 + \rho_1^2)(\rho_2^2 - p^2) + \frac{\rho_2^2}{\rho_1^2}(\lambda^2 + \rho_2^2)(\rho_1^2 - p^2) \right]$$
$$+ \frac{pD\rho_2}{\rho_1} \cos(\theta_1 + \theta_2)(\lambda^2 + \rho_2^2)(\rho_1^2 - p^2)$$

In an unfavourable case we might have $x_1 = y_1 = 2$; $p = \frac{1}{2}$, $\rho_1 = 2\sqrt{2}$, $\rho_2 = 3\sqrt{2}$, $D = \sqrt{2}$; the ratio of the last two terms in Q is then at most

$$4 \cdot \tfrac{9}{4} : \tfrac{1}{2}\sqrt{2} \cdot \tfrac{3}{2} \text{ or nearly } 8 : 1.$$

We are therefore justified in neglecting the last term as well as p^2 in the first two terms; and a table of values of P and Q′ where

$$Q' = \rho_1^2 \rho_2^2 (2\lambda^2 + \rho_1^2 + \rho_2^2)$$

was constructed.

The coefficient of Q′ is $-2 \sin \alpha \cos \alpha \frac{x_1 y_1}{\rho_1^2}$ and is always positive; the value of $x_1 y_1 / \rho_1^2$ or $\frac{1}{2} \sin 2\theta_1$ can easily be read off to two decimal places by placing a piece of tracing paper on the chart having lines ruled, along which $\frac{1}{2} \sin 2\theta_1$ has the values 0·1 to 0·5.

The weight of an equation is inversely proportional to the square of the mean error of its absolute term, and in the present case the weight is therefore proportional to

$$\frac{\lambda^2 D^2}{\lambda^2 P + 2 \sin \alpha \cos \alpha \frac{x_1 y_1}{\rho_1^2} Q'}$$

which is the simplest form to which the expression for the weight can be reduced. The numerical value of D is found by means of a foot rule, P and Q′ are taken together out of the table; $x_1 y_1 / \rho_1^2$ is read off graphically, and $2 \sin \alpha \cos \alpha$ is twice the product of the coefficients of x and y. No attention has to be paid to signs. For convenience the weights are multiplied by 100.

The table of P and Q′ which was actually used is here given; the values of Q′ are printed in italics immediately beneath the corresponding values of P.

Let g be the weight of an equation (m in number),
v be a residual—*i.e.* the perpendicular from the deduced radiant on the meteor's path.

Then the mean error of an absolute term whose weight is unity is

$$\sqrt{\frac{[gvv]}{m-2}}.$$

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TABLE OF P AND Q′.

$P = \rho_1^2(1 + \rho_2^2) + \rho_2^2(1 + \rho_1^2)$ and $Q' = \rho_1^2\rho_2^2(1 + \rho_1^2 + 1 + \rho_2^2)$

Neglect p when $p < ·05$
$\rho_1 > ·80$

r_1 → r_2 ↓	·05	·10	·15	·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70	·75	·80	·85	·90
·10	·01	↓	·15															
	...																	
·15	·02	·03		·20														
	...	...																
·20	·04	·05	·06		·25													
	...	...	...															
·25	·06	·07	·09	·10		·30												
	...	...	...	...														
·30	·09	·10	·11	·13	·15	·18	·35											
	...	...	...	...	*·01*	...												
·35	·12	·13	·14	·16	·20	·23		·40										
	...	...	...	*·01*	*·02*	*·02*												
·40	...	·17	·18	·20	·24	·27	·32		·45									
			...	*·01*	*·02*	*·03*	*·04*											
·45			·22	·25	·28	·33	·36	·42		·50								
			...	*·02*	*·03*	*·04*	*·06*	*·07*										
·50	·50	→		·30	·33	·38	·43	·49	·55		·55							
				·02	*·03*	*·05*	*·07*	*·10*	*·12*									
·55		·55			·40	·45	·50	·56	·62	·71		·60						
					·04	*·06*	*·09*	*·12*	*·15*	*·19*								
·60			·60			·51	·56	·64	·70	·79	·89		·65					
						·08	*·11*	*·15*	*·18*	*·23*	*·29*							
·65				·65		·57	·64	·72	·78	·88	·97			·70				
							·13	*·17*	*·22*	*·28*	*·35*							
·70					·70			·81	·88	·98	1·09	1·19			·75			
								·21	*·26*	*·34*	*·42*	*·48*						
·75						·75		·89	·98	1·09	1·20	1·32	1·45	1·60		·80		
								·25	*·31*	*·39*	*·49*	*·58*	*·71*	...				
·80							·80		1·08	1·21	1·32	1·46	1·60	1·75			·85	
									·36	*·46*	*·56*	*·69*	*·83*	*·98*				
·85	p = ·06, subtract ·01 from P							·85	1·20	1·33	1·46	1·60	1·74	1·91	2·08			·90
									·43	*·52*	*·66*	*·80*	*·95*	*1·13*	*1·33*			
·90	p = ·07, ” ·01 ”								·90	1·44	1·59	1·75	1·91	2·10	2·27	2·49	2·70	↓
										·59	*·76*	*·92*	*1·11*	*1·31*	*1·54*	*1·79*	*2·05*	
·95	p = ·08, ” ·01 ”									·95	→	1·90	2·08	2·27	2·46	2·69	2·93	
												1·05	*1·26*	*1·49*	*1·75*	*2·04*	*2·35*	
1·00	p = ·09, ” ·02 ”										1·00	2·05	2·26	2·47	2·68	2·92	3·16	
												1·18	*1·45*	*1·71*	*2·00*	*2·33*	*2·69*	
1·05	p = ·10, ” ·02 ”										1·05	2·21	2·44	2·67	2·90	3·14	3·40	
												1·33	*1·64*	*1·94*	*2·26*	*2·63*	*3·04*	
1·10	p = ·11, ” ·02 ”										1·10			2·88	3·13	3·39	3·67	3·98
														2·19	*2·56*	*2·98*	*3·44*	*3·94*
1·15	p = ·12, ” ·03 ”											1·15		3·09	3·36	3·64	3·94	4·27
														2·43	*2·88*	*3·35*	*3·85*	*4·42*
1·20	p = ·13, ” ·03 ”												1·20		3·60	3·92	4·24	4·59
															3·20	*3·76*	*4·33*	*4·97*
1·25	p = ·14, ” ·04 ”													1·25		4·20	4·52	4·89
																4·19	*4·84*	*5·51*
1·30	p = ·15, ” ·04 ”													1·30			4·85	5·24
																	5·38	*6·16*
1·35	p = ·16, ” ·05 ”													1·35			5·16	5·57
																	5·95	*6·81*
1·40														1·40				5·95
																		7·58
1·45														1·45				6·31
																		8·35
1·50																		

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The mean error of an absolute term of weight g, or of an observed p, is therefore

$$\frac{1}{\sqrt{g}}\sqrt{\frac{[gvv]}{m-2}}.$$

But we have just found that this was ϵ_p, where

$$\epsilon_p^2 = \frac{\epsilon^2}{2}\frac{1}{\lambda^2 D^2}\left\{\lambda^2 P + 2 \sin a \cos a \frac{x_1 y_1}{\rho_1^2} Q'\right\}$$

and we have taken

$$g = 100 \frac{\lambda^2 D^2}{\lambda^2 P^2 + 2 \sin a \cos a \frac{x_1 y_1}{\rho_1^2} Q}.$$

Hence

$$\frac{1}{\sqrt{g}}\sqrt{\frac{[gvv]}{m-2}} = \epsilon_p = \frac{\epsilon}{\sqrt{2}}\frac{10}{\sqrt{g}}, \text{ or } \epsilon = \tfrac{1}{10}\sqrt{\frac{2}{m-2}[gvv]}.$$

Now g is of -2 dimensions in length, and ϵ is therefore a number, and is the circular measure of the arc of great circle, which is the mean error of an observed beginning or end of a meteor's path.

The effect of zenith attraction, rotation and orbital motion of the Earth have been neglected as inappreciable. For let ξ, η be the coordinates of the apparent radiant referred to parallel axes through the unknown true radiant; let x, y be the coordinates of the true radiant. Then ξ, η are known at any moment, when the character of the swarm is known. The perpendicular distance from the point $x+\xi$, $y+\eta$ on to the line of a meteor's track is

$$(x+\xi)\cos a + (y+\eta)\sin a - p = 0.$$

The error committed in neglecting ξ, η is therefore

$$dp = \xi \cos a + \eta \sin a,$$

of which the maximum value is $\sqrt{\xi^2+\eta^2}$, when $\tan a = \eta/\xi$. From Dr. Kleiber's table in *Monthly Notices*, lii. p. 352, we find that, to reduce observations made on a given night to the midnight of that night, we must apply the following corrections:—

	$\Delta\alpha$	$\Delta\delta$	ξ	η	$\sqrt{\xi^2+\eta^2}$
	°	°			
9 P.M.	−0·40	+0·71	−·38	+1·24	1·3
Midnight	−0·71	+0·49	−·67	+ ·89	1·1
3 A.M.	−0·63	+0·24	−·59	+ ·42	0·7

The unit of p was for convenience always taken as $\frac{1}{10}$ inch; x, y, ξ, η are also expressed in terms of this unit. As the probable error of x and y generally amounts to about ±0·5, it will be quite

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sufficient to apply the mean correction $\xi = -0{\cdot}5$, $\eta = +1{\cdot}0$ to the values of x and y, deduced from meteors fairly evenly distributed in time from 9 P.M. to 3 A.M., in order to get the absolute place at midnight. Had these disturbing causes produced appreciable effects, the path of the apparent radiant should be drawn round the origin as true radiant, and p measured from the point on this curve corresponding to the time of observation instead of from the origin. In the case of the *Perseids* this point does not appreciably shift during the night.

The observations discussed so far are :—

		Meteors.	
W. F. Denning at	Bristol	149	Communicated in manuscript.
M. Morine	Poulkova	49	"Sur les Perséids, observées en 1890." (*Brédikhine, Bull. St. Pét. Acad.* 1894, No 1.)
M. Maggi	Volpeglino	46	*Pub. Osservatorio di Brera*, No. 7, p. 58.
M. Scharbe	Dorpat	43	*Astr. Nach.* Bd. 149, p. 213.
M. Lewitzky	Dorpat	23	,, ,, ,, ,,
	Total ...	310	

Mr. Denning very kindly copied out some of his observations of *Perseids* and sent them to me at my request. I am much indebted to him for having done this.

The results from these observations are as follows :—

Observer.	Date. August.	r. Probable Error of p of weight unity.	r. Expressed in Arc.	w_0. Mean weight of one Observation.	ρ. Probable Error of Beginning and End.	n. Number of Meteors.	Radiant. R.A. 45°+.	Decl. 57°+.
Denning	10, 1877	8·41	± 4°·82	7·9	± 0°·68	24	− 3°·74 ± 0°·94	+ 0°·74 ± 0
,,	10, 1878	7·61	4·36	6·3	0·62	20	+ 0·70 ± 1·20	− 0·63 ± 0
,,	9, 1880	6·63	3·80	5·9	0·54	13	+ 2·12 ± 1·25	+ 0·61 ± 0
,,	9, 1893	2·58	1·48	5·0	0·21	16	− 1·96 ± 0·47	− 0·21 ± 0
,,	10, 1896	4·40	2·52	5·3	0·36	29	+ 0·06 ± 0·53	+ 0·52 ± 0
,,	11, 1898	2·72	1·56	5·7	0·22	32	+ 0·16 ± 0·26	+ 1·15 ± 0
,,	10, 1899	2·18	1·25	7·2	0·21	15	− 1·28 ± 0·29	− 0·34 ± 0
Morine	9, 1893	6·38	3·66	5·2	0·51	23	− 5·08 ± 0·74	− 1·30 ± 0
,,	11, 1893	3·70	2·12	5·4	0·29	26	+ 2·60 ± 0·44	+ 1·22 ± 0
Maggi	10, 1872	19·96	11·44	15·8	1·62	46	+ 0·45 ± 0·96	+ 0·90 ± 0
Scharbe	9, 1898	5·31	3·04	2·1	0·43	17	+ 4·01 ± 1·52	+ 2·26 ± 0
,,	11, 1898	5·24	3·00	2·2	0·42	26	− 2·22 ± 1·28	+ 1·78 ± 0
Lewitzky	9, 1898	7·17	4·11	8·7	0·57	11	− 3·63 ± 0·97	− 0·38 ± 0
,,	11, 1898	8·36	4·79	6·4	0·67	12	− 3·85 ± 1·32	+ 0·64 ± 1

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The fourth column gives the value of the probable error of p in arc of great circle; if a small circle is drawn round the radiant with this value of r as radius, the prolongation of a meteor's path backwards to the radiant would be just as likely to pass without this circle as within, if the observation were of weight unity. If this is divided by the square root of w_0, we get the mean radius of this circle for each observer. It will be noticed that the mean weight of one of Maggi's observations is more than twice that of any of the other observers; the reason is that the former registers his meteor paths as much longer than most observers. But in spite of their great weight his observations have rather a large probable error, which may be accounted for by his having been influenced by the position of stars near to the meteor's path, for most of his meteors begin and end at a star. Of the 46 meteors observed by him, there are only 29 distinct paths; in one case 6 meteors are recorded as pursuing identically the same path. At the time of making these observations Maggi was a practised observer, having recorded paths for several years previously. Morine had observed at Poulkova for two or three years before; but I do not know how far back Lewitzky and Scharbe's observations extend. Mr. Denning seems to have become more skilled with practice, and his probable error is now very small.

But it must be remembered that there is mixed up with the probable error here found a possible real diffuseness of the radiant, and it is therefore desirable to discuss observations of other showers; but it is not easy to find a large enough number of meteor-paths all observed on *one* night to give a reliable probable error. However, these observations of Mr. Denning's seem to bear out his belief that radiant areas as such do not really exist (*Monthly Notices*, vol. xlv. p. 96), or at any rate that the radiant area is very small, which is what one would rather expect.

That the large difference between Morine's probable error on August 9 and 11 of the same year is possibly due to real differences of radiant is confirmed by the observation of another Poulkova observer, Stratonof; a chart of the meteors shows it at once. Mr. Denning's observations on the first of these nights do not confirm the Poulkova observers, his probable error being very small.

If, then, the radiant of the *Perseids* has a very small area of diffuseness, these observations show that a first-class observer is as likely as not to be in error by a quarter of a degree in recording the beginning or end of a meteor's path, whilst an observer of less extensive experience may err by half a degree. The resulting radiants will have probable errors of rather less than a quarter of a degree and of nearly three-quarters respectively.

In the opinion of Mr. Denning, three meteors will often give as good and certain a radiant as 30, especially when they are well-observed paths from a radiant of low altitude. Let us suppose then that we have 4 *Perseids* each of weight 6 and one in each quadrant and that they are inclined at 45° to the rectangular

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axes, so that x and y are equally well determined. They furnish the following equations :—

$$\begin{array}{rl} \cdot 71x + \cdot 71y - p_1 = 0. & \text{Wt. } 6. \\ \cdot 71x - \cdot 71y - p_2 = 0. & 6. \\ -\cdot 71x + \cdot 71y - p_3 = 0. & 6. \\ -\cdot 71x - \cdot 71y - p_4 = 0. & 6. \end{array}$$

Whence the weights of x and y are both 12 ; if we take the probable error, r, of one observation of weight unity as $\pm 6\cdot 0$ we have

$$r_x = \frac{r}{\sqrt{p_x}} = \pm 1\cdot 73 \qquad r_y = \frac{r}{\sqrt{p_y}} = \pm 1\cdot 73$$

and therefore the probable error of the deduced radiant is

$\pm 1^\circ\cdot 8$ in R.A. and $\pm 1^\circ\cdot 0$ in Declination.

A first-class observer might have $r = \pm 2\cdot 5$, in which case the probable errors would be

$\pm 0^\circ\cdot 8$ in R.A. and $\pm 0^\circ\cdot 4$ in Declination.

But it will only be very rarely that four meteors can be observed as conveniently situated as those four ideal ones, and thus one coordinate is sure to be more strongly determined than the other. And, again, if the radiant is a point and the observer's r is known from previous observation, the more meteors that are made use of the greater will be the weight of x and y and the smaller the resulting probable error of the radiant. It seems, then, that to get a reliable determination of a radiant, as many meteors as possible should be observed and used in the reduction of their radiant point.

I have not yet been able to reduce the observations made on other days than August 9, 10, and 11, in order to discuss the motion of the radiant. The mean position of the radiants of all the observers on those three days are :—

	R.A.	Declination.	Longitude.	Latitude.
Aug. 9	42°·77 ± 0°·34	+ 56°·94 ± 0°·19	59°·25 ± 0°·24	38°·42 ± 3°·19
10	44·01 ± 0·23	+ 56·94 ± 0·14	60·05 ± 0·16	38·15 ± 0·14
11	45·58 ± 0·22	+ 58·22 ± 0·15	61 67 ± 0·15	39·07 ± 0·15

Summary of Results.

1. In deducing the radiant point, attention ought to be paid to the weights of the individual meteors.

2. An expression is therefore found, in a form capable of practical application, for assigning the proper weights.

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3. It seems reasonable to assume that all meteors belonging to the same shower have nearly the same *real* length of path; the apparent length varies with the distance from the radiant. Since then an inordinately great or small weight—about 5 times or $\frac{1}{5}$ the mean weight for a given observer—indicates that the path of the meteor is not of a length proportionate to its distance from the radiant, it follows that the weights are to a certain extent a criterion of a meteor's right to be included in the determination of the radiant. But Mr. Denning's words, "radiant points should be determined by the observer of the meteors," must be remembered; the calculator must only include meteors which the observer gives him labelled as *Perseids*.

4. The effect of zenith attraction, rotation, and orbital motion of the Earth on one night's observations may be neglected as insensible in the case of the *Perseids*.

5. Assuming the radiant of this shower to be a point—

A. The probable error of a recorded beginning or end of a meteor's path varies so far as direction of path is concerned from ±0°·2 to ±1°·6.

B. To get a reliable determination of the radiant the more meteors made use of the better.

C. The probable error of a radiant derived from about 20 to 30 meteors varies from ±0°·2 to ±1°·0.

6. Assuming the radiant to be an area—

A. There are indications of a change in size of this area.

7. A discussion of observations of meteors belonging to other strong showers would decide the question of how much of the probable error here found is due to error of observation, and how much to real diffuseness of radiant.

8. The position of a radiant point derived from Mr. Denning's observations may be relied upon to within three-quarters of a degree of great circle, and I think that the present discussion, so far as it goes, makes it clear that eye-observations are of sufficient accuracy to show the existence of stationary radiants. Very great care must be taken in combining meteors to form a radiant, and if there is any doubt as to the reality of stationary radiants it is on this point, and not on the accuracy of the record of path, that it rests.

Leonids observed at Cambridge Observatory, 1900 *November* 13, 14, 15. By J. C. W. Herschel, B.A.

(*Communicated by Sir Robert Ball.*)

[The arrangement for observing the *Leonids* in 1900 was entrusted to Mr. J. C. W. Herschel, Research Student at the Observatory. He was assisted by the following members of the